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[In the previously printed solution nothing is given to show *how* the *literal* terms on right of (*a*), (*b*), (*c*) arise.]

In case of friction of the movable pulleys, account generally would have to be taken of the *vis viva* of rotary motion of $P_1$ and $P_2$, there being, then, two additional terms in the energy equation $T = V$, namely,

$$\frac{k_1^2}{2} P_1\dot{\varphi}_1^2 \quad \text{and} \quad \frac{k_2^2}{2} P_2\dot{\varphi}_2^2,$$

$k_1$ and $k_2$ being the radii of gyration of the pulleys $P_1$, $P_2$, supposed to be cylinders of radii, say, $r_1$, $r_2$, and so $k_1^2 = r_1^2/2$, $k_2^2 = r_2^2/2$, and $\dot{\varphi}_1$, $\dot{\varphi}_2$, the angular velocities of $P_1$, $P_2$.

There were not stated any dimensions of $P_1$, $P_2$; permitting us to take $r_1$ and $r_2$ indefinitely small. This would, though, be a special condition.

In taking $r_1$, $r_2$ of finite value, the energy equation would be

$$T = \tfrac{1}{2}(m_1 + P_1 + m_3 + P_2 + m_4 + m_5)\dot{x}_1^2 + \text{etc.} + (-m_3 + P_2 + m_4 + m_5)\dot{x}_1\dot{x}_2 + \text{etc.}$$
$$+ \tfrac{1}{2}k_1^2\dot{\varphi}_1^2 + \tfrac{1}{2}k_2^2\dot{\varphi}_2^2 = m_1gx_1 + \text{etc.} + C = V. \qquad (d)$$

Regardless of signs, $r_1\,\dot{\varphi}_1 = \dot{x}_1$, $r_2\,\varphi_2 = x_2$; and substituting $\dot{\varphi}_1$, $\dot{\varphi}_2$ from these equations in (*d*), we have an equation differing in form from (*i*), but to which (*ii*) may be applied as before, and generally with different values for $\ddot{x}_1$ etc.

So far the fixed pulley has been thought of as smooth. If this pulley be regarded as rough, the energy equation would still further be modified so as to take account of work done against friction, or of energy due to its rotation and some assigned mass

**350 (Mechanics). Proposed by J. B. REYNOLDS, Lehigh University.**

If an elastic tube filled with liquid under pressure doubles in length in what ratio will the radius be increased?

### SOLUTION BY THE PROPOSER.

Let the initial length and radius of the tube be $l_0$ and $r_0$, the final lengths $l$ and $r$ and the coefficient of elasticity of a strip of the material one unit wide be $\lambda$.

If $P$ is the pressure per square unit we have, since the length is doubled,

$$P\pi r^2 = 2\pi r\frac{\lambda}{l_0}l_0; \qquad \text{whence} \qquad P = \frac{2\lambda}{r}. \qquad (1)$$

Again, if $T$ is the peripheral tension per unit length, in order to balance the internal pressure

$$T = \lim_{\Delta\theta \doteq 0}\frac{Pr\Delta\theta}{2\sin\Delta\theta/2} = Pr. \qquad (2)$$

Also,

$$T = \frac{\lambda}{2\pi r_0}(2\pi r - 2\pi r_0) = \frac{\lambda}{r_0}(r - r_0). \qquad (3)$$

By (1), (2), and (3) we get,

$$Pr = 2\lambda = \frac{\lambda}{r_0}(r - r_0);$$

whence $r = 3r_0$.

**356 (Mechanics). Proposed by FLORENCE P. LEWIS, Goucher College, Baltimore, Md.**

A ray of light enters a prism having vertex angle $\alpha$. If the angle between the incoming and outgoing directions is defined as the angle of deviation, at what angle must the ray enter the prism in order that the angle of deviation may be a minimum?

### SOLUTION BY J. B. REYNOLDS, Lehigh University.

We have

$$\sin\varphi = \epsilon\sin x, \qquad \sin\psi = \epsilon\sin(\alpha - x),$$

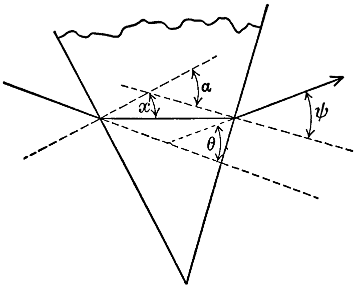
$\epsilon$ being the index of refraction. If $\theta$ is the angle of deviation, we have, by geometry, $\theta = \psi + \varphi - \alpha$ or

$$\theta = \sin^{-1}\epsilon\sin(\alpha - x) + \sin^{-1}\epsilon\sin x - \alpha.$$

For $\theta$ to be a minimum

$$\frac{d\theta}{dx} = \frac{-\epsilon\cos(\alpha - x)}{\sqrt{1 - \epsilon^2\sin^2(\alpha - x)}} + \frac{\epsilon\cos x}{\sqrt{1 - \epsilon^2\sin^2 x}} = 0;$$

from which $x = \frac{1}{2}\alpha$ and therefore $\varphi = \psi = \sin^{-1}\epsilon\sin\frac{1}{2}\alpha$. That is, the angle of deflection or deviation is a minimum when the ray in the glass makes equal angles with the faces of the prism.



**357 (Mechanics). Proposed by J. B. REYNOLDS, Lehigh University, South Bethlehem, Pa.**

Two beads each of mass $m$ connected by a string of length $2l$ and carrying a mass $m'$ at its middle point are threaded symmetrically with respect to the major axis which is vertical on a smooth ellipse of eccentricity $e$ and latus rectum $2l$. The string is held taut and horizontal, then released; find the velocities of the beads when the end ones impinge.

## SOLUTION BY THE PROPOSER.

There are two cases: I, when the end beads are at the extremities of the upper latus rectum II, when the end beads are at the extremities of the lower latus rectum. In either case when the end beads impinge their velocities will be equal and the velocity of $m'$ will be zero. If $a$ is the semi-major axis we have by the principle of work,

Case I

$$m'g\{l - a(1 - e)\} - 2mga(1 - e) = \frac{2m}{2}v^2;$$

or since

$$l = a(1 - e^2),$$

$$mv^2 = ag(1 - e)(m'e - 2m);$$

or

$$v^2 = \frac{gl}{m(1 + e)}\{m'e - 2m\}.$$

For the beads to impinge in this case $m' > 2m/e$.

Case II

$$m'g\{l + a(1 - e)\} + 2mga(1 - e) = \frac{2m}{2}v^2,$$

whence as before

$$v^2 = \frac{gl}{m(1 + e)}\{(2 + e)m' + 2m\}.$$

**260 (Number Theory). Proposed by ALBERT A. BENNETT, University of Texas.**

Let $\binom{n}{r}$ denote, as usual, the binomial coefficient $n!/[r!(n - r)!]$, where $\binom{n}{0} = 1$, but where $n$, $r$, $(n - r)$ are always to be supposed to be positive integers or zero. Let us define $k_i(m, n)$ as $\Sigma_i \binom{m-i+j}{i-j}\binom{n-j}{j}$. Prove that the following recursion formula is consistent:

$$\Sigma(-1)^i k_i(m, n)C_{m+n-i} = \binom{m+n}{m}$$

and determine $C_0 = 1, C_1 = 1, C_2 = 2, C_3 = 5, C_4 = 14, C_5 = 42, C_6 = 132, C_7 = 429, C_8 = 1{,}430$, etc. Prove also that these quantities satisfy the following relations, as well:

$$\sum_i (-1)^i C_{m-n-i}\binom{m-1}{i} = 0$$

for each $n$ where $2n \leq m$.